Classical simulations of quantum circuits

Resource-theoretic approach to quantum computing

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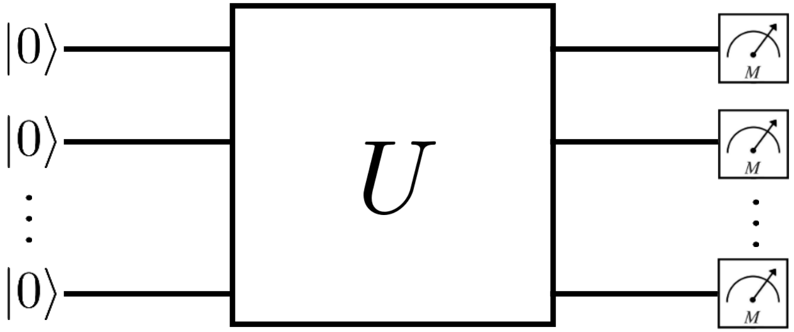
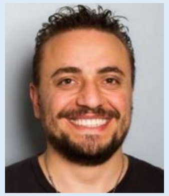
TEAM-NET

Outline

1. Motivation
2. Background
3. Simulating Clifford + T circuits
4. Unified simulation framework
5. Outlook



In collaboration with:



H. Pashayan

S. Bartlett

THE UNIVERSITY OF SYDNEY

Motivation

Foundations

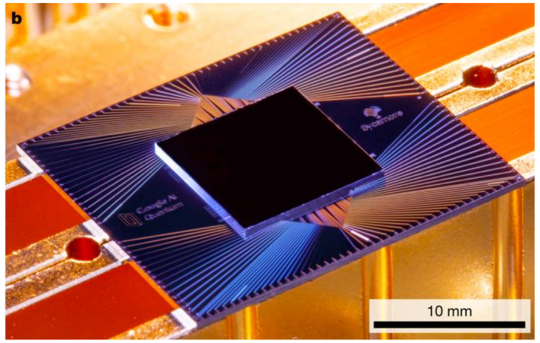
Strong evidence that quantum computing is more powerful than classical computing.

What component of quantum theory is responsible for this quantum speed-up?

- Entanglement?
- Coherence?
- Contextuality?
- Wigner negativity?
- Special combination of the above?

Applications

Characterization, verification, and validation of near-term quantum devices



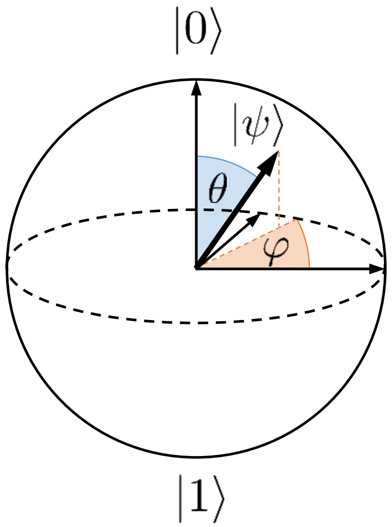
Outline

1 bit: 1 of 2 states $\{0, 1\}$

1 qubit: linear combination of 2 basis states $\{|0\rangle, |1\rangle\}$

$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ Probability of measuring $|i\rangle$: $p_i = |c_i|^2$

Useful parametrisation: $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$



n bits: 1 of 2^n states $\{0, 1\}^{\times n}$

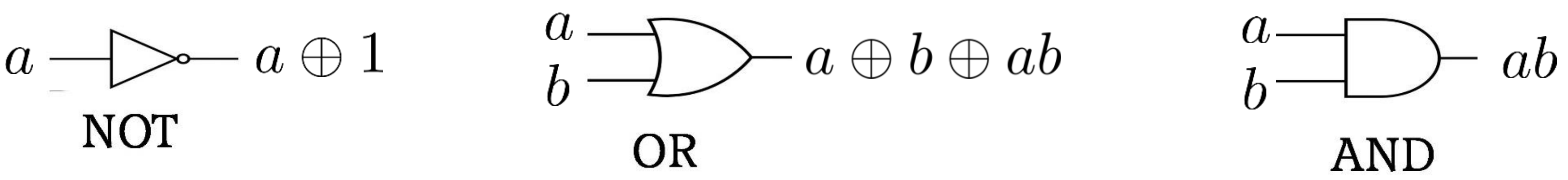
E.g. 01 or 11

n qubits: linear combination of 2^n basis states $\{|0\rangle, |1\rangle\}^{\otimes n}$

E.g. $|\Psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$

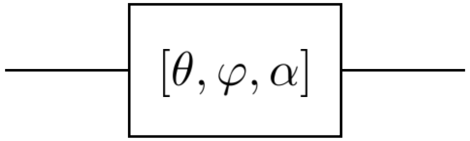
Background:
Universal sets of (quantum) gates

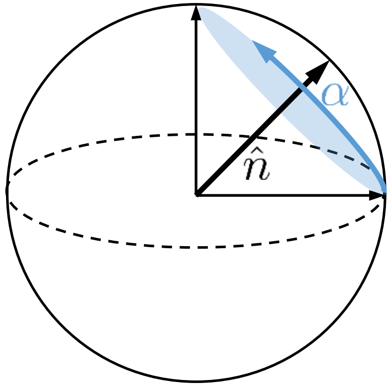
Classical gate: mapping between n and m bits



Quantum gate: unitary matrix U transforming a state vector $|\psi\rangle$ to $U|\psi\rangle$

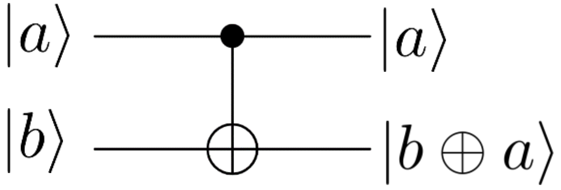
General 1-qubit gate



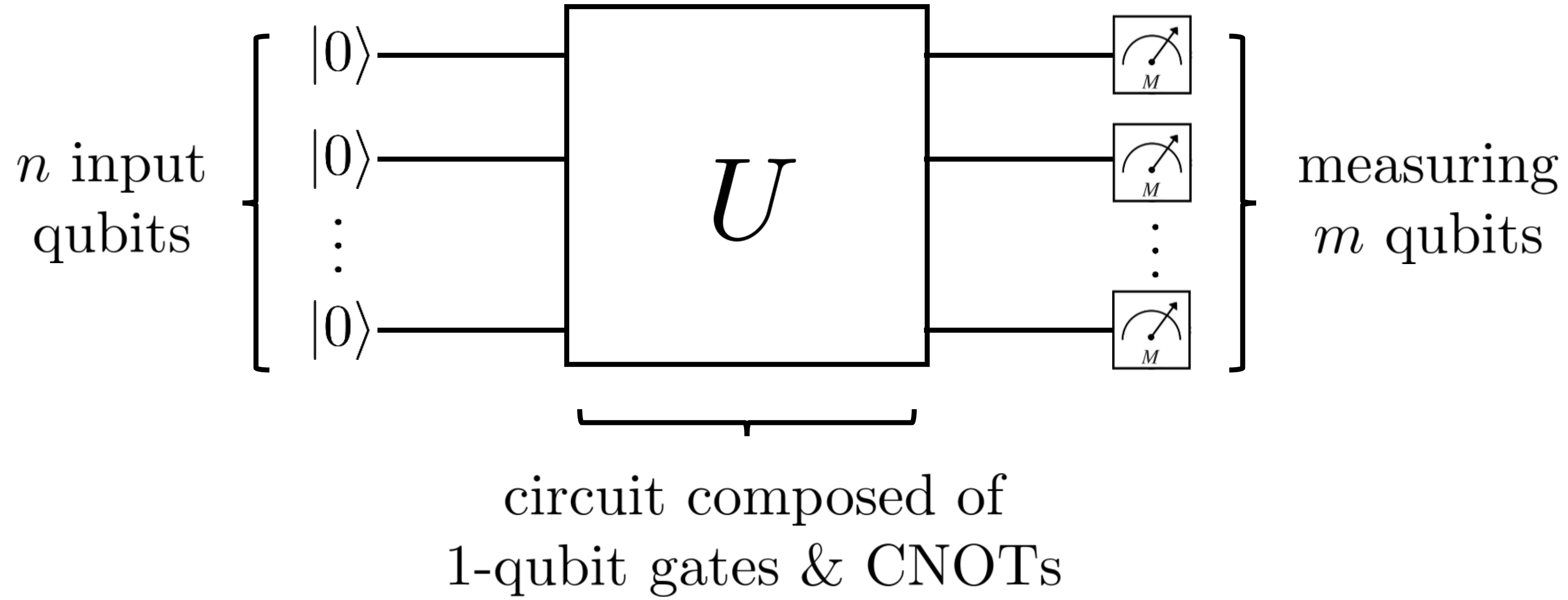


Rotation around axis $\hat{n} = (\theta, \varphi)$ by angle α

2-qubit gate: CNOT



E.g. $|00\rangle + |11\rangle \xrightarrow{CNOT} |00\rangle + |10\rangle$



Prob. of measuring qubit 1 in state $s_1, \ldots,$ qubit m in state s_m:

$$p_U(\boldsymbol{s}) = \| \langle s_1 s_2 \ldots s_m | U | 0_1 0_2 \ldots 0_n \rangle \|_2^2$$

Strong simulation

Calculate $p_U(\boldsymbol{s})$

Weak simulation

Sample from $p_U(\boldsymbol{s})$

Our simulation

Estimate $p_U(\boldsymbol{s})$

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1. Motivation
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3. Simulating Clifford + T circuits
    a. Pauli gates and stabiliser states
    b. Clifford gates and Gottesmann-Knill
    c. Step 1: Gadgetizing T gates
    d. Step 2: Stabilizer decompositon
    e. Step 3: Sampling stabilizers
    f. Step 4: Fast norm estimation
4. Unified simulation framework
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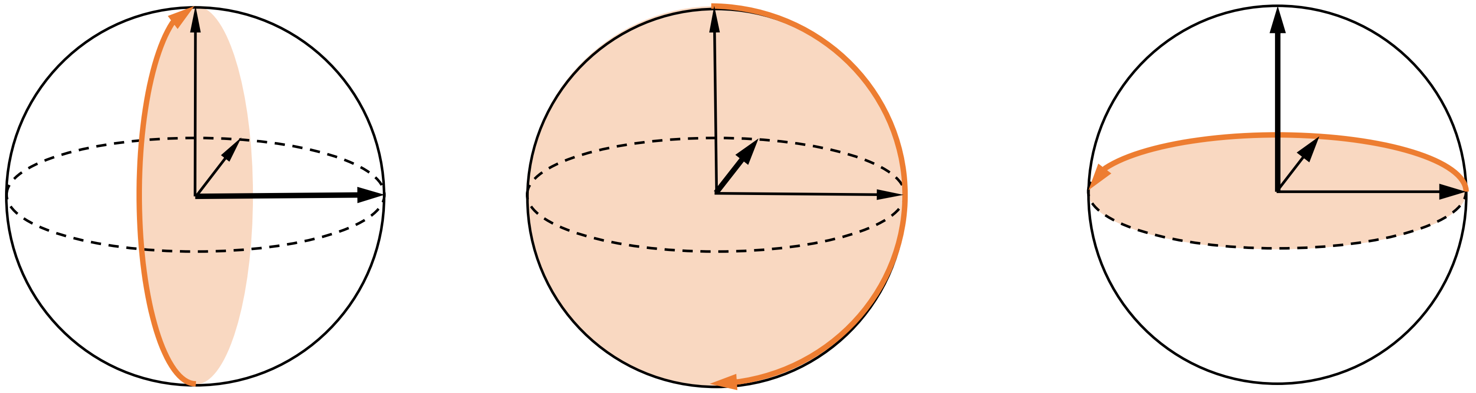
Simulating Clifford + T circuits
Pauli gates and stabiliser states

1-qubit Pauli gates:



$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

***n*-qubit Pauli gates:** E.g. $\sigma_x \otimes \sigma_z \otimes \mathbb{1} \otimes \sigma_y$ (only ± 1 eigenstates)

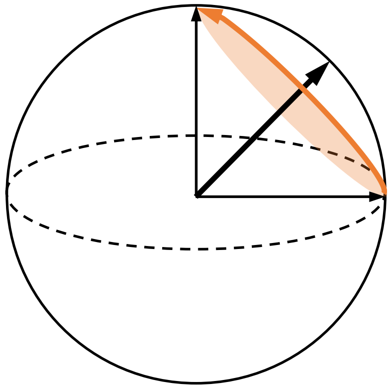
***n*-qubit stabilizer state:** simultaneous eigenstate of n commuting Pauli matrices

$$\text{E.g. } |0\rangle \leftrightarrow \{\sigma_z\} \text{ or } |00\rangle + |11\rangle \leftrightarrow \{\sigma_z \otimes \sigma_z, \sigma_x \otimes \sigma_x\}$$

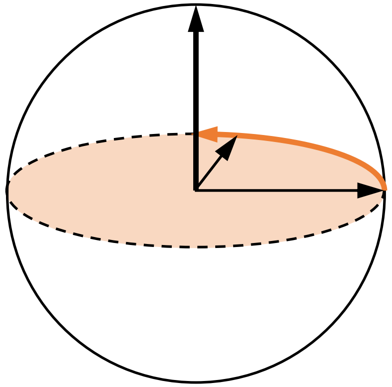
Simulating Clifford + T circuits
Clifford gates and Gottesmann-Knill theorem

***n*-qubit Clifford gate C**: for a Pauli operator P, $CPC^\dagger$ also a Pauli operator

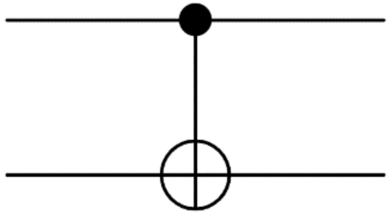
Generators:



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$



$$S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$$



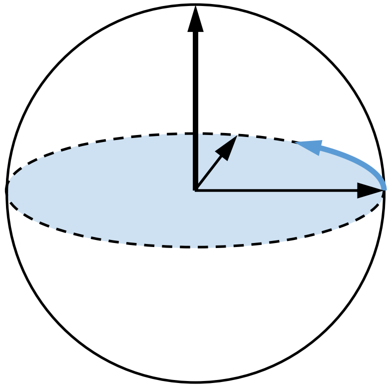
CNOT

Gottesmann-Knill theorem: evolution of stabiliser states through Clifford circuits can be efficiently described on a classical computer.

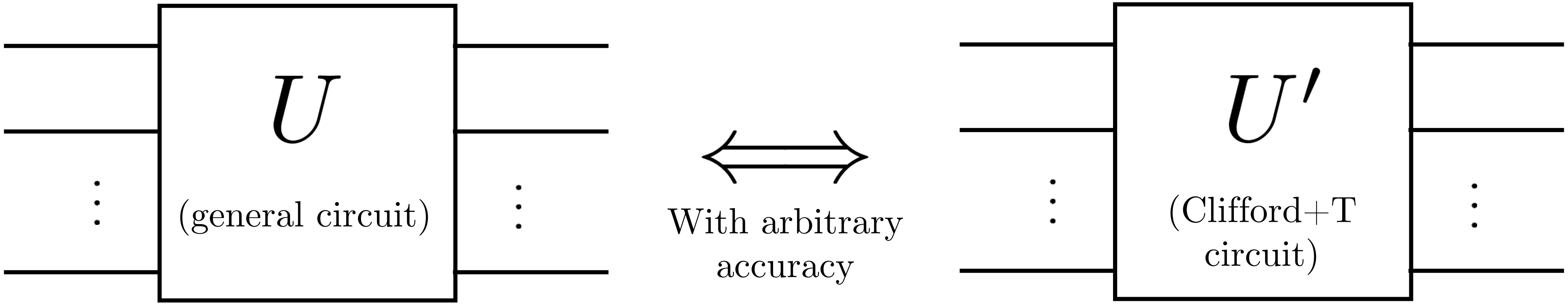
(n-qubit stabiliser state described by n Pauli operators, each of them is mapped by a Clifford gate to another Pauli operator. Just keep track of stabilisers.)

Clifford gates are not universal!

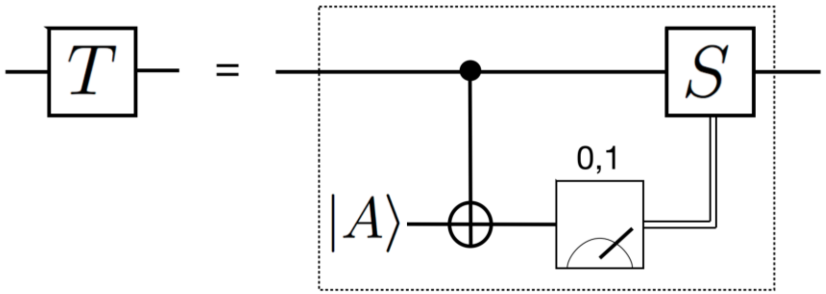
Adding a single T gate is enough!



$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

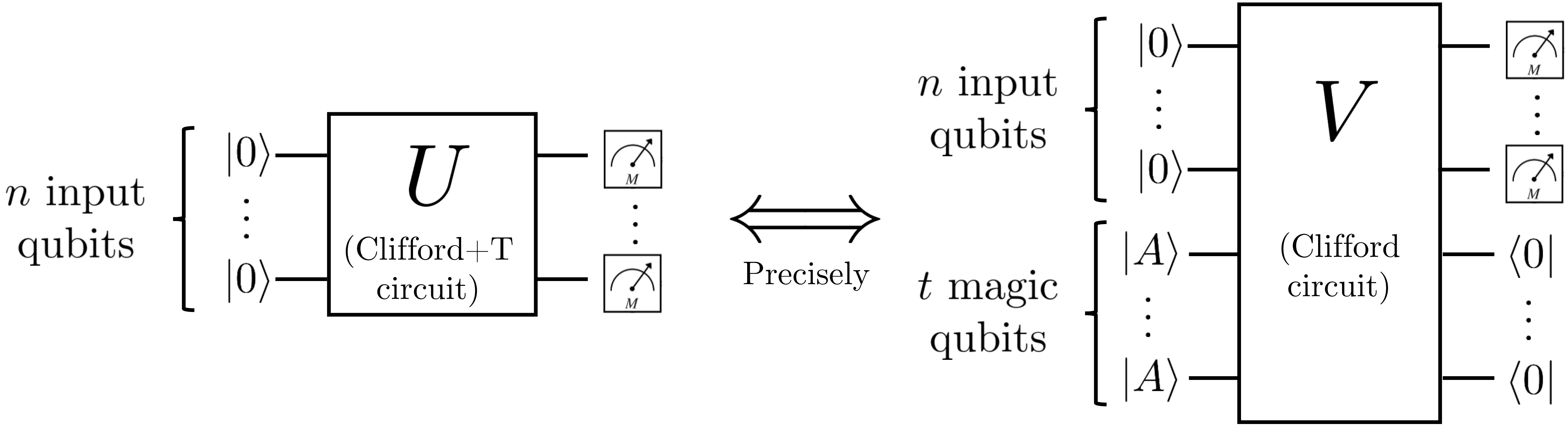


Step 1: Gadgetizing T gates with magic states



$$|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle)$$

arXiv:1601.07601



Simulating Clifford + T circuits
Step 2: Stabilizer decomposition of magic states

Non-unique stabilizer decomposition of $|A\rangle$

$$|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle) = \alpha|\tilde{0}\rangle + \alpha^*|\tilde{1}\rangle \qquad \alpha = \frac{1+i(\sqrt{2}-1)}{2}$$

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$ Stabilizer state stabilized by σ_x

$$|\tilde{1}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$ Stabilizer state stabilized by σ_y

t magic qubits $\left[\begin{array}{l} |A\rangle — \\ \vdots \\ |A\rangle — \end{array}\right.$ $\Longleftrightarrow$

$$|A\rangle^{\otimes t} = (\alpha|\tilde{0}\rangle + \alpha^*|\tilde{1}\rangle)^{\otimes t} = \sum_{\boldsymbol{a}\in\{0,1\}^{\times t}} \alpha^{t-|\boldsymbol{a}|}(\alpha^*)^{|\boldsymbol{a}|}|\tilde{\boldsymbol{a}}\rangle$$

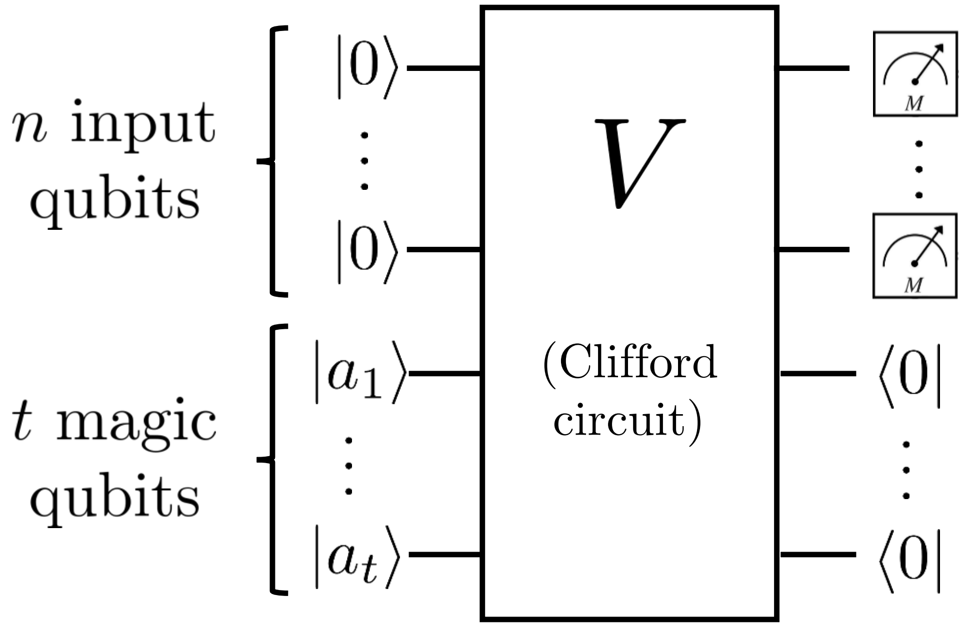
Due to linearity may evolve each stabilizer term separately.
But there are 2^t terms!

Step 3: Sampling from stabilizer decomposition

Instead of computing each of 2^t stabilizer terms $|\tilde{a}\rangle$ we will:

- Uniformly sample $2^{\gamma t}$ terms $|\tilde{a}\rangle$ with $\gamma \approx 0.228$
- Use Gottesmann-Knill to evolve each term
- Project $(n+t)$-qubit stabilizer to obtain $(n-m)$-qubit unnormalized stabilizer

arXiv:1601.07601



Why this value of γ? $\gamma = \log_2(|\alpha| + |\alpha^*|)^2$

Why $2^{\gamma t}$ is enough? Hoeffding's inequality.

Simulating Clifford + T circuits
Step 4: Fast norm estimation

We are left with $S = 2^{\gamma t}$ unnormalized stabilizer states $|\Psi_i\rangle$

Length of the sample average is the estimated probability:

$$p = \left\| \frac{1}{S} \sum_i |\Psi_i\rangle \right\|^2$$

Employ the efficient stabilizer norm estimation from arXiv:1601.07601

Final run-time of the algorithm: $\tau_{\text{exp}} \sim \tilde{O}\left(2^{\gamma t} t^3 \epsilon_{\text{tot}}^{-4}\right)$

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Unified simulation framework

Various splittings into free (efficiently simulable) theory and resourceful (exponentially hard to simulate) operations:

- Clifford + T gates
- Gaussian gates + Non-gaussian gate
- Matchgate circuits + SWAP gate
- ...

Gadgetization

Decomposition of resource states into free states

Sampling from free-state decomposition

Estimating probability

Outlook

New **Quantum Resource Group** established at Jagiellonian University
(leader + 2 post-docs + 2 PhD students + MSc student)

Objective 1: A unified framework for classical simulations of quantum circuits

1. Developing a unified scheme for classical simulation of universal quantum circuits based on a three-step algorithm.
2. Devising novel algorithms with improved run-time scaling by employing alternative free element decompositions (e.g. pure free states). Implementing these algorithms on classical computers and employing them to certify and verify NISQ devices.
3. Investigating the interconversion problem for the resource theory of magic states.